Wax

19687

ARPA Network Information Center
Stanford Research Institute
Menlo Park, California 94025

A. K. Jain (Collins)

25 October 1973

Packet Radio Temporary Note #76

NIC #19687

"AUTOCORRELATION IN THE PRESENCE OF DETERMINISTIC DISTURBANCES USING FFT"

1.0 INTRODUCTION

One purpose of this note is to document the results of the investigation undertaken to study the effects of the channel filters on the theoretical performance of a matched filter SAWD (Surface Acoustic Wave Device) in terms of the degradation of its autocorrelation peak and side lobes. A computer program using a FFT (Fast Fourier Transform) subroutine has been developed for this purpose.

However, the general purpose of this note is to record our ability to correlate almost any deterministic input with the specified impulse response of a matched filter using the Fast Fourier Transform technique. The program with reasonable manipulations should be valuable in analyzing the performance of both spread and unspread systems when the signal is subjected to some deterministic interference, and also corrupted by bandpass amplitude and phase distortions through filters, antenna resonances, echoes from mis-matched transmission lines, etc.

RECEIVED AT NIC OCTOBER 31, 1973.

2.0 DESCRIPTION OF THE COMPUTER PROGRAM

The present structure of the computer program "AUTCOR" is best understood by referring to Figure 1. The bi-phase modulated input signal is formulized based on reading a specific length code sequence (up to 127 chips length). This sequence forms the basis of the impulse response of the matched filter as well as the basis of the 2-bit long (both bits identical) input signal. The program is normalized for a carrier frequency of 7 Hz and a chip rate of 1 Hz. This allows 7 carrier cycles (of sinewaves) for each chip interval, and may, of course, be altered if desired.

As shown in Figure 1, the input signal may be filtered, or added with a disturbance before correlating it with the matched SAWD. The various combinations of the available options are obvious from the figure. The label "channel filters" refers to any filter(s) encountered by the input signal at the transmit or the receive (or both) ends. In the present structure of the program, the channel filters consist of a normalized 2.05 Hz wide* 2-pole Chebychev transmit filter, and a normalized 2.55 Hz wide 3-pole Chebychev receive filter. An additional filter shown in the figure is a normalized 2.5 Hz wide 3-pole Butterworth filter. These filters can, of course, be changed by defining a new transfer function. Only the low pass equivalent transfer functions of the filters need be defined as the program automatically converts to the normalized bandpass transfer function.

The program automatically chooses the required number of samples for the FFT subroutine based on the code length and the carrier frequency. The processing time to compute one fourier transform, or the inverse fourier transform, based on a certain number of samples N is directly proportional to $N \log_2 N$. In order to conserve the overall processing time as well as the main core storage of a typical modern computer, each bit period of the correlating signals is divided into 4 sections, each section is sequentially correlated with its appropriate counterparts, and the sectionalized results are finally summed using what is

*3 db bandwidth

usually referred to as the overlap-add technique. The program results into "Calcomp" plot of the correlated output in the form of a continuous waveform.

3.0 FILTERING EFFECTS ON CORRELATOR'S PERFORMANCE

In order to conserve the processing time, much of the investigation was first conducted based on 7-chip codes. Although the investigation of the filtering effects has been extended up to 127-chip codes, all of the results of 7, 31, and 127 chip length codes to this effect are documented in this section.

In order to understand the results, the following plots need to be reviewed with reference to the figures stated.

a. Autocorrelation of 7-chip maximal length code (code sequence is 1110010) with no filtering (Figure 2)
b. Correlation of the 7-chip code with the input signal filtered through the following:
   (i) 2-pole Chebychev 2.05 Hz wide filter (Figure 3)
   (ii) 3-pole Chebychev 2.55 Hz wide filter (Figure 4)
   (iii) 3-pole Butterworth 2.5 Hz wide filter (Figure 5)
   (iv) The above three filters in series (Figure 6)
c. Autocorrelation of 31 chip maximal length code (code sequence: 11111,00011,01110,10100,00100,10110,0) with no filtering (Figure 7)
d. Correlation of the 31 chip code with the input signal filtered through the following:
   (i) 2-pole Chebychev 2.05 Hz wide filter (Figure 8)
   (ii) 3-pole Chebychev 2.55 Hz wide filter (Figure 9)
   (iii) 3-pole Butterworth 2.5 Hz wide filter (Figure 10)
   (iv) The above three in series (Figure 11)

usually referred to as the overlap-and technique. The program results into "clumpy" plot of the correlated output in the form of a continuous waveform.

3.0 FILTERING EFFECTS ON CORRELATOR'S PERFORMANCE

In order to conserve the processing time, much of the investigation was first conducted based on 7-chip codes. Although the investigation of the filtering effects has been extended up to 127-chip codes, all of the results of 7, 31, and 127 chip length codes to this effect are documented in this section.

In order to understand the results, the following plots need to be reviewed with reference to the figures stated.

a. Autocorrelation of 7-chip maximal length code (code sequence is 1110010) with no filtering (Figure 2)

b. Correlation of the 7-chip code with the input signal filtered through the following:

(i) 2-pole Chebychev 2.05 Hz wide filter (Figure 3)

(ii) 3-pole Chebychev 2.55 Hz wide filter (Figure 4)

(iii) 3-pole Butterworth 2.5 Hz wide filter (Figure 5)

(iv) The above three filters in series (Figure 6)

c. Autocorrelation of 31 chip maximal length code (code sequence: 11111,00011,01110,10100,00100,10110,0) with no filtering (Figure 7)

d. Correlation of the 31 chip code with the input signal filtered through the following:

(i) 2-pole Chebychev 2.05 Hz wide filter (Figure 8)

(ii) 3-pole Chebychev 2.55 Hz wide filter (Figure 9)

(iii) 3-pole Butterworth 2.5 Hz wide filter (Figure 10)

(iv) The above three in series (Figure 11)

e. Autocorrelation of 127-chip maximal length code (code sequence: 11111,11011,10110,11110,10001,01100,10111, 11000,10000,00110,01101,10001,11001,11010,11100,00100, 11000,00101,01011,01001,00101,00111,10010,00110,10100, 00) with no filtering (Figure 12)

f. Correlation of the 127-chip code with the input signal filtered through all three filters listed previously (Figure 13)

Based on the above figures, the following observations (Table 1) are made:

	AUTOCORRELATION PEAK / MAX SIDELOBE* (DB)				
Code Length	No Filter	2 Pole Cheb.	3 Pole Cheb.	3 Pole Butt.	All Three Filters
7	$\frac{7}{1}$ (16.9)	$\frac{5.3}{1.7}$ (9.8)	$\frac{5.3}{1.8}$ (9.4)	$\frac{5.3}{1.8}$ (9.4)	$\frac{3.5}{1.5}$ (7.4)
31	$\frac{31}{5}$ (15.9)	$\frac{25}{5}$ (14)	$\frac{24}{5}$ (13.6)	$\frac{24}{5}$ (13.6)	$\frac{19}{4.4}$ (12.7)
127	$\frac{127}{8}$ (24)	NA	NA	NA	$\frac{78}{8}$ (19.8)

*listed for the first cycle of the response

TABLE 1

It is apparent from the above data that the peak autocorrelation is reduced by about 40 to 50% depending on the spread factor; however, the maximum side lobe level of the first cycle of response remains relatively constant, particularly for the larger spreads. It should also be apparent from Figures 2 through 13 that the second cycle side lobe level which is normally supposed to be 1 for the maximal length code also increases with input filters.

Code Length	AUTOCORRELATION PEAK / MAX SIDELOBE (DB)				
	No Filter	2 Pole Cheb.	3 Pole Cheb.	3 Pole Butt.	All Three Filters
7	$\frac{7}{1}$ (16.9)	$\frac{5.3}{1.8}$ (9.8)	$\frac{5.3}{1.8}$ (9.4)	$\frac{5.3}{1.8}$ (9.4)	$\frac{3.5}{1.3}$ (7.4)
31	$\frac{31}{5}$ (15.9)	$\frac{25}{5}$ (14)	$\frac{25}{5}$ (13.6)	$\frac{24}{5}$ (13.6)	$\frac{19}{5}$ (12.7)
127	$\frac{127}{5}$ (24)	NA	NA	NA	$\frac{79}{5}$ (19.8)

4.0 ADDITIONAL SCOPE OF THE PROGRAM

As introduced earlier in this note, there are additional areas where this program can perhaps be of value in obtaining some quantitative answers as well as some qualitative insight in terms of what to expect whenever the signal is corrupted by means of a defined phenomenon. In order to bring this scope in its proper perspective, some additional results are outlined in this section. However, only the 7-chip code is used for such investigation for the reasons already stated.

4.1 Response to a Rectangular Pulse

This preliminary investigation is conducted with the response to the ignition spikes in mind. A one chip wide rectangular pulse with unit height (same as the signal amplitude) was first sent through a 3-pole 2.5 Hz normalized width, 7 Hz center frequency, Butterworth filter. The response of this filter to the rectangular pulse as given by the program is plotted in Figure 14. Notice, as expected, there are two distinct cycles in this response, one relating to the pulse going from zero to unity, and the other a chip later (1 second for 1 Hz chip rate) relating to the pulse going from unity to zero.

This bandpass response is then correlated (without the presence of the input signal) with the impulse response of the matched filter for various fractional chip widths of the input pulse. Figures 15 through 18 are the plots of correlated response to pulse width varying from 0.7 chip wide to 1 chip wide. Note that in each case there is an identical pulse appearing at the input at the start of the second bit interval to meet the present requirement of the program that the first two bits be specified at the input.

In summary, however, a unit high pulse varying in widths from 0.7 to 1 times the chip interval, and with 7 carrier cycles per chip interval, results into maximum correlated peak to peak amplitudes of the order of approximately 0.8 to 1.

Digitized by the Internet Archive
in 2025 with funding from
Amateur Radio Digital Communications, Grant 151

https://archive.org/details/temporary-note-076

4.2 Response to Additive CW Interference

Figures 19 through 23, present the autocorrelation of the 7 chip codes at normalized 7 Hz carrier frequency and 1 Hz chip rate with CW signals starting in phase with the input signal, with CW frequency varied from 6.5 to 6.9 Hz. As apparent from the plots, for in phase CW signals the worst case side lobes occur with the CW frequency closest to the signal carrier frequency.

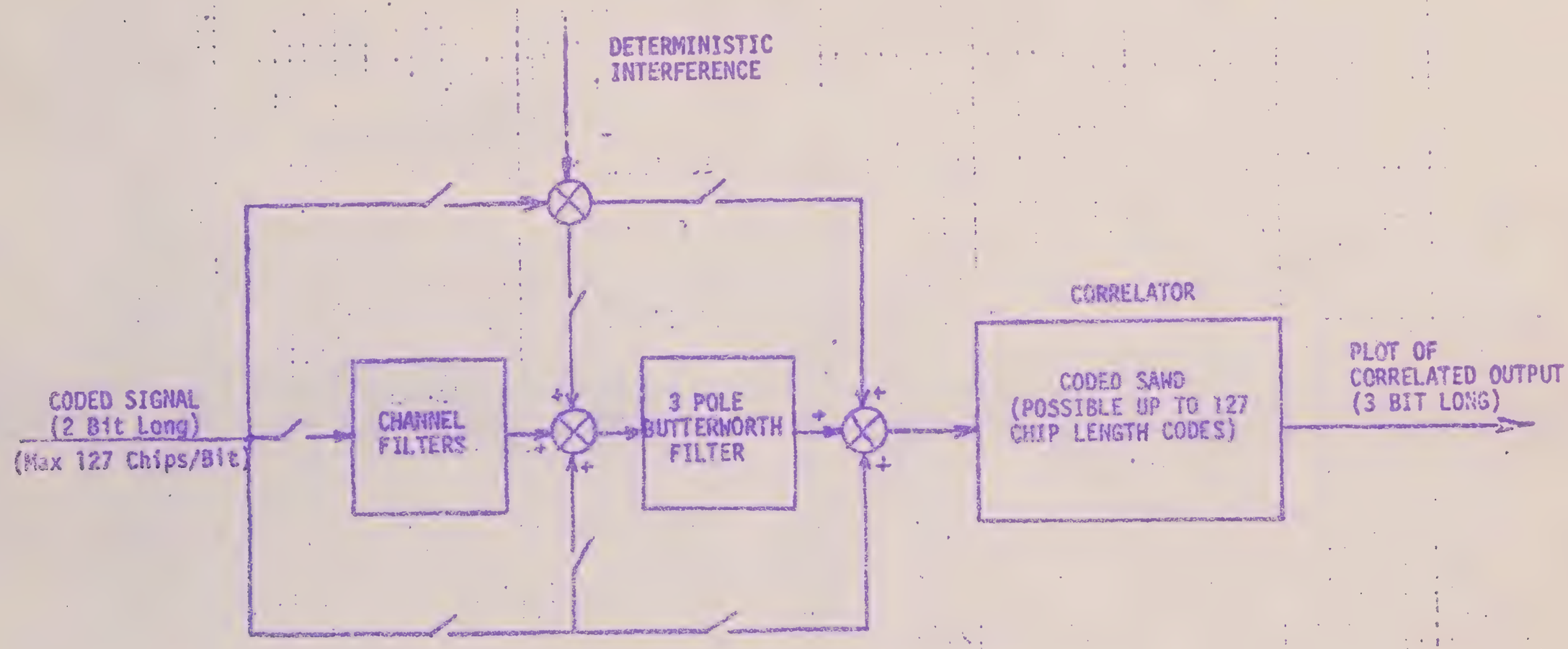
5.0 ADDITIONAL COMMENTS AND CONCLUSIONS

a. As confirmed from the results of Section 3.0, it would be desirable to avoid any filtering. Therefore, enough consideration should be given to using MSK (Minimum Shift Keying) that has the advantages of constant amplitude and minimum phase shift. Consequently, some hardware impacts of MSK relating to the feasibility of obtaining compatible SAWDS should be evaluated. This may be a subject of a later note.

b. As should perhaps be obvious from the description of the program, signals up to approximately 1000 chip length codes can be processed using this program provided we work with baseband signals only. In fact, the most of the results documented thus far in this note could have been obtained with baseband signals; however, the bandpass approach has put forth the versatility of the program in proper perspective. Note that for baseband analyses the normalized carrier frequency equals 1 Hz.

c. In order to get the most practical benefit from this analytical tool, an attempt will be made to model the SAWD based on its design parameters so that the theoretical impulse response of the matched filter can be scaled down to somewhat of a real impulse response that must result due to imperfections in the device manufacturing.

d. In addition to the modeling of SAWD itself, an attempt will be made to include the effect of the matching network associated with the SAWDS. Measurements will also be made on the SAWDS to determine how well the analytical results agree with the devices purchased.



Note: The program is normalized for a carrier frequency of 7 Hz and chip rate of 1 Hz. Program requires only the low pass transfer function of filters which is automatically converted to the normalized bandpass transfer function.

FIGURE 1

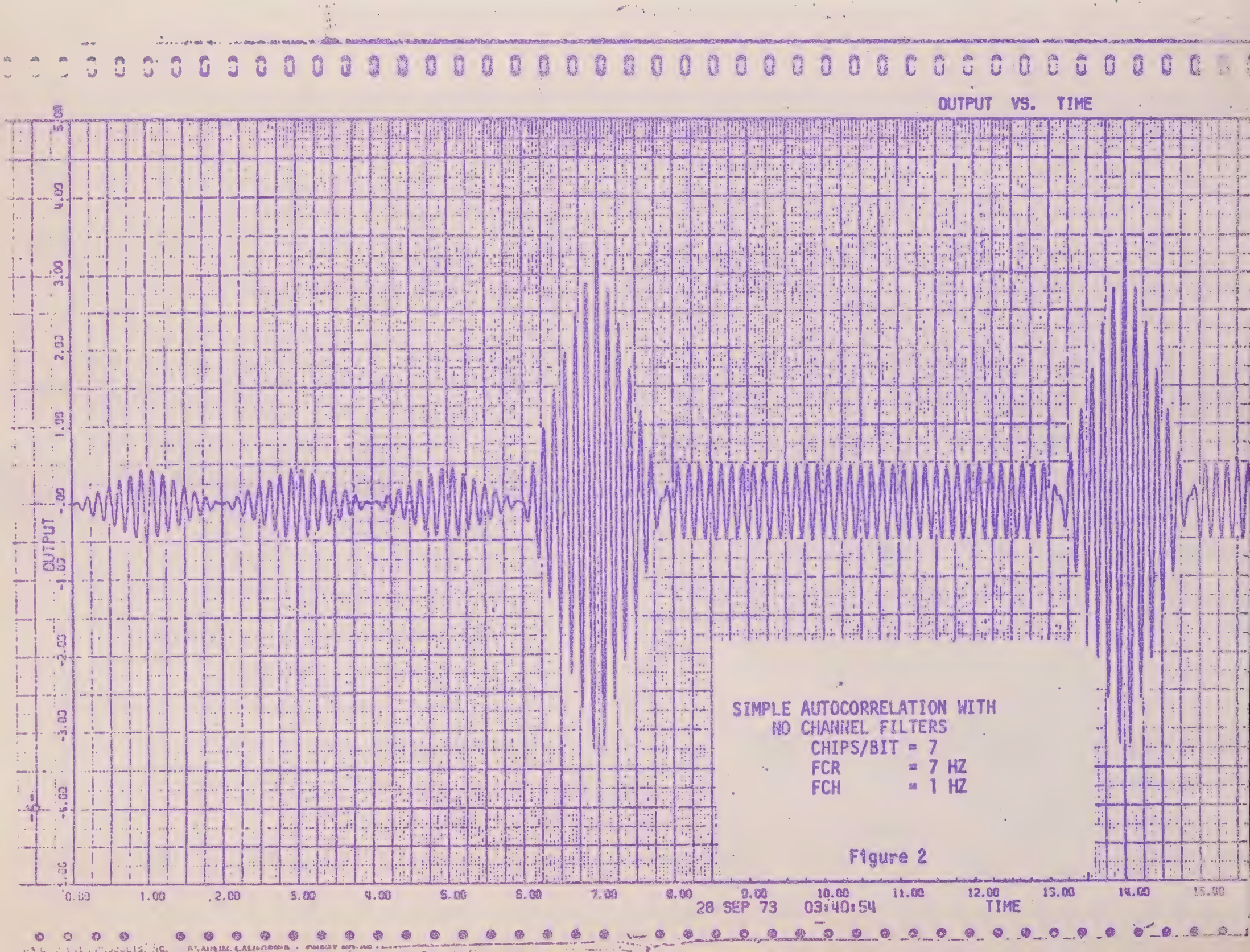

OUTPUT VS. TIME
OUTPUT
5.00
4.00
3.00
2.00
1.00
-.00
-1.00
-2.00
-3.00
-4.00
-5.00
0.00
1.00
2.00
3.00
4.00
5.00
6.00
7.00
8.00
9.00
10.00
11.00
12.00
13.00
14.00
15.00
28 SEP 73 03:40:54
TIME
SIMPLE AUTOCORRELATION WITH
NO CHANNEL FILTERS
CHIPS/BIT = 7
FCR = 7 HZ
FCH = 1 HZ

Figure 2

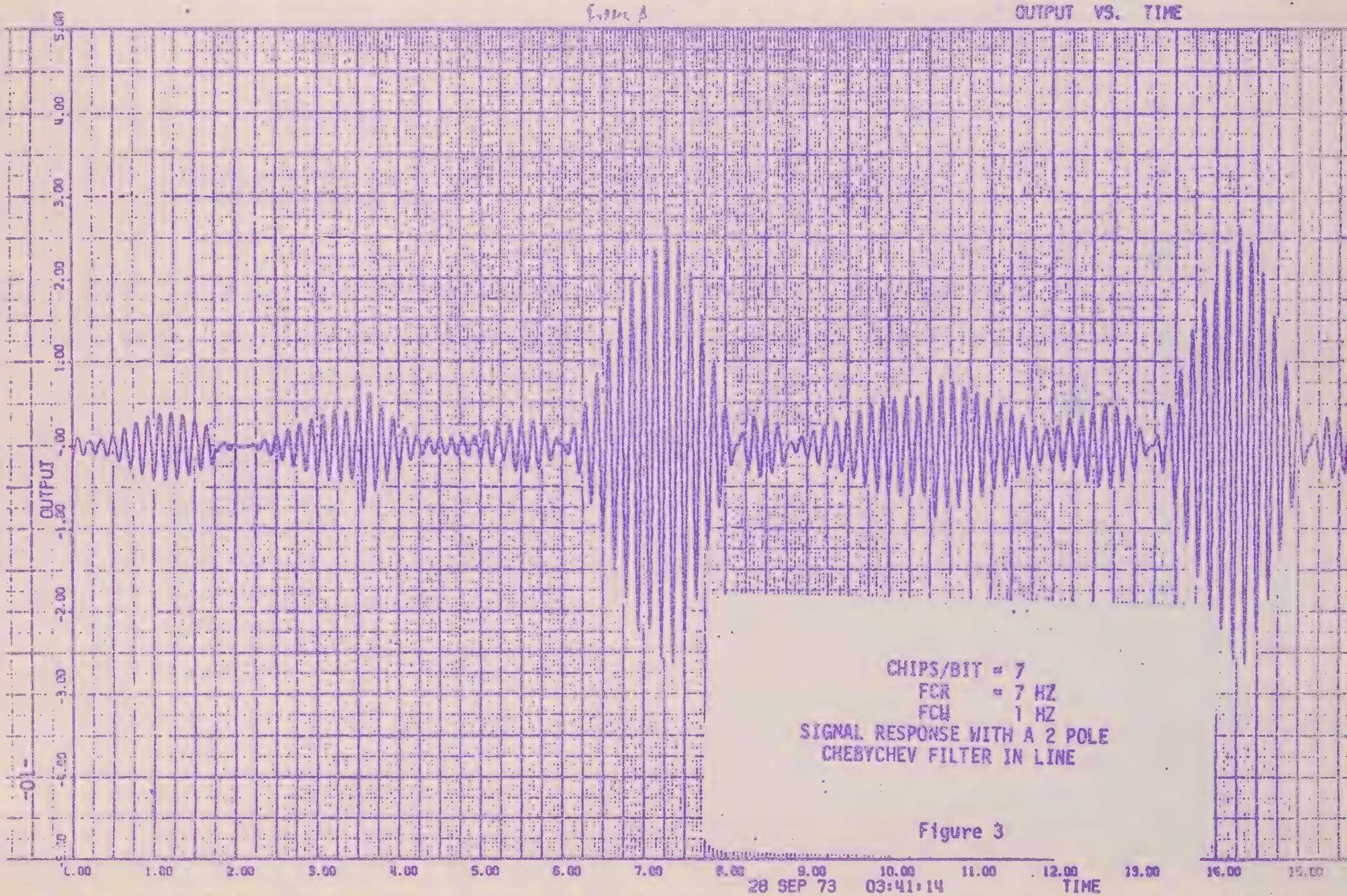

OUTPUT VS. TIME
OUTPUT
5.00
4.00
3.00
2.00
1.00
-.00
-1.00
-2.00
-3.00
-4.00
-5.00
1.00
2.00
3.00
4.00
5.00
6.00
7.00
8.00
9.00
10.00
11.00
12.00
13.00
14.00
15.00
28 SEP 73 03:41:14
TIME
CHIPS/BIT = 7
FCR = 7 HZ
FCW 1 HZ
SIGNAL RESPONSE WITH A 2 POLE
CHEBYCHEV FILTER IN LINE

Figure 3

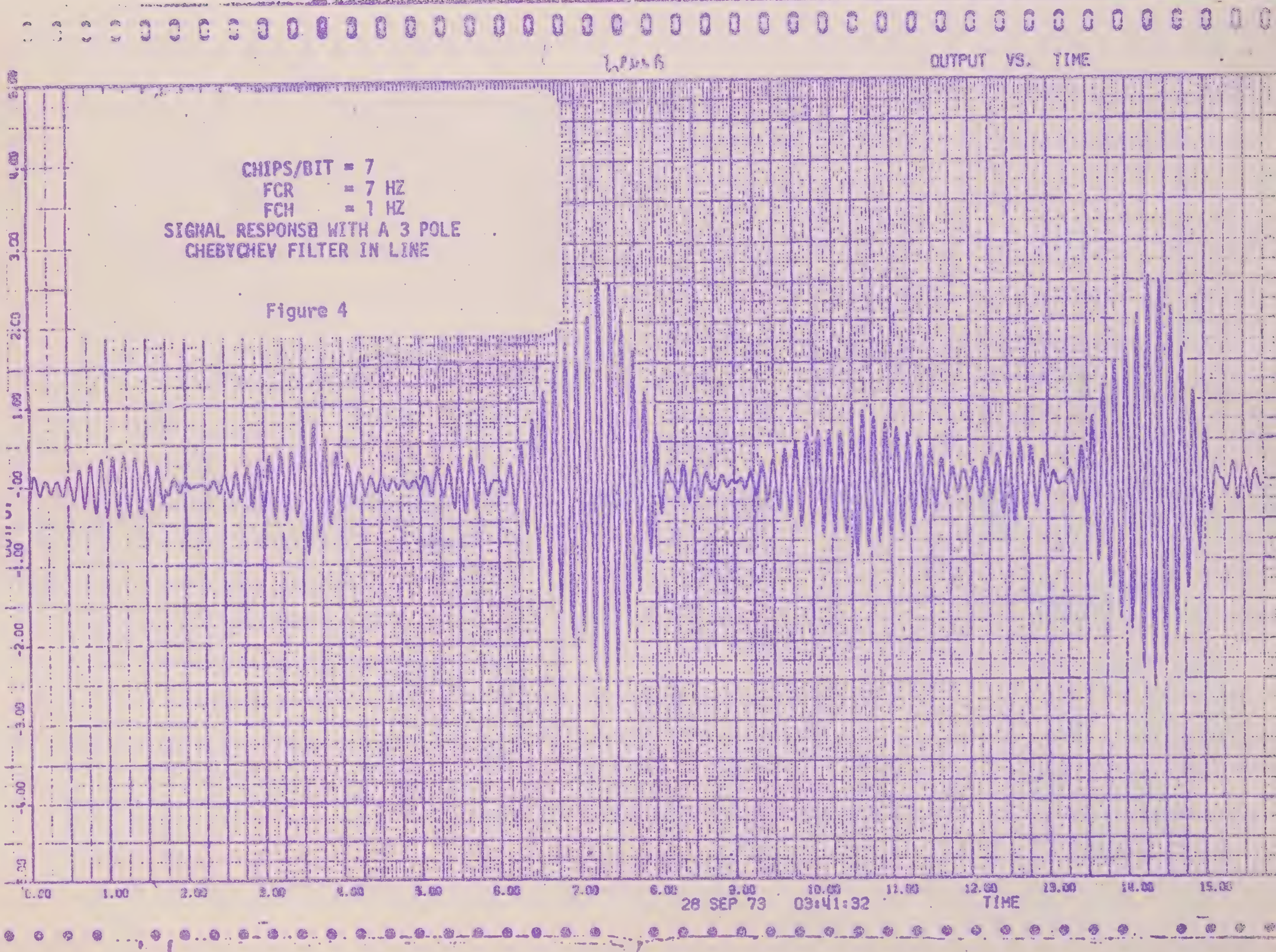
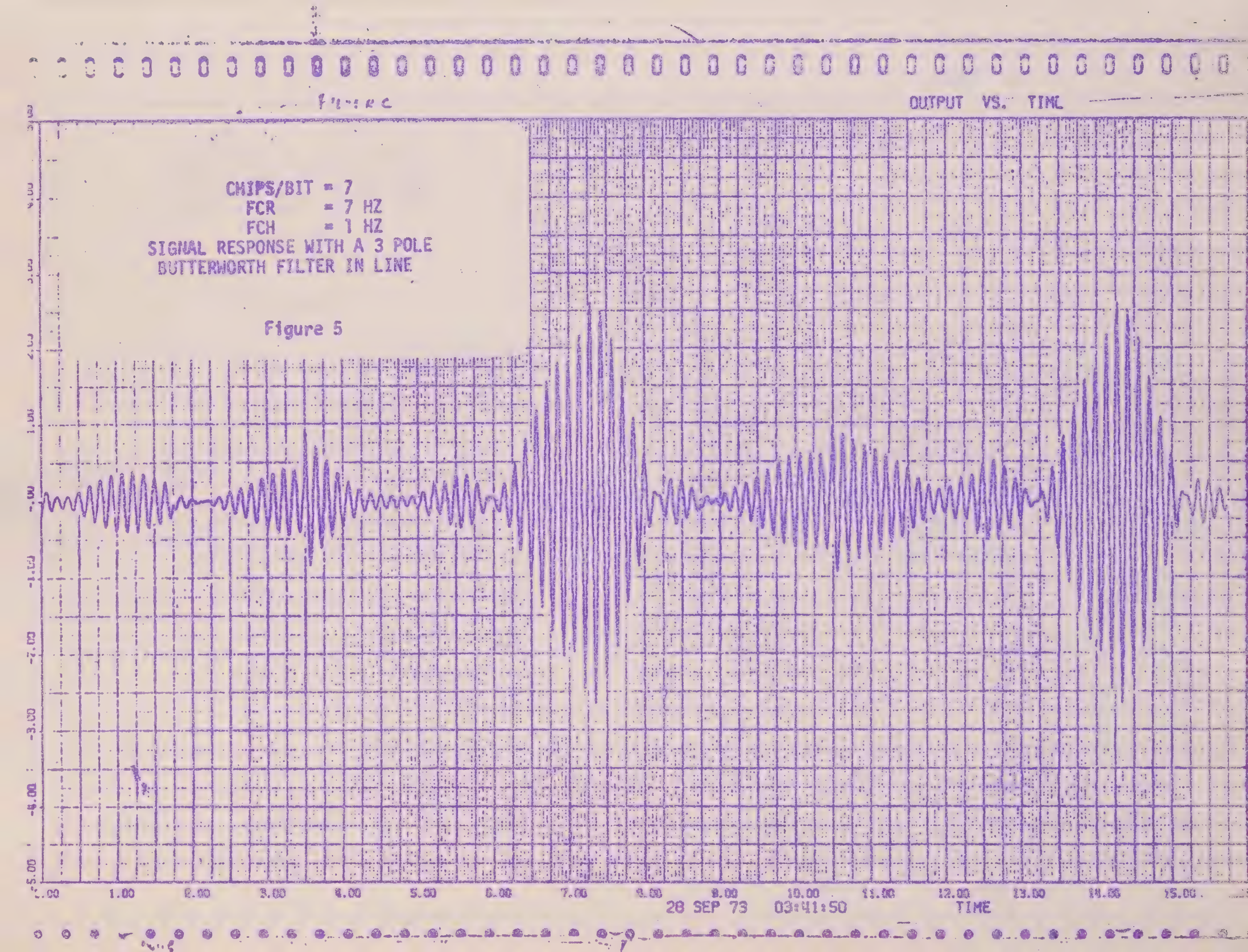
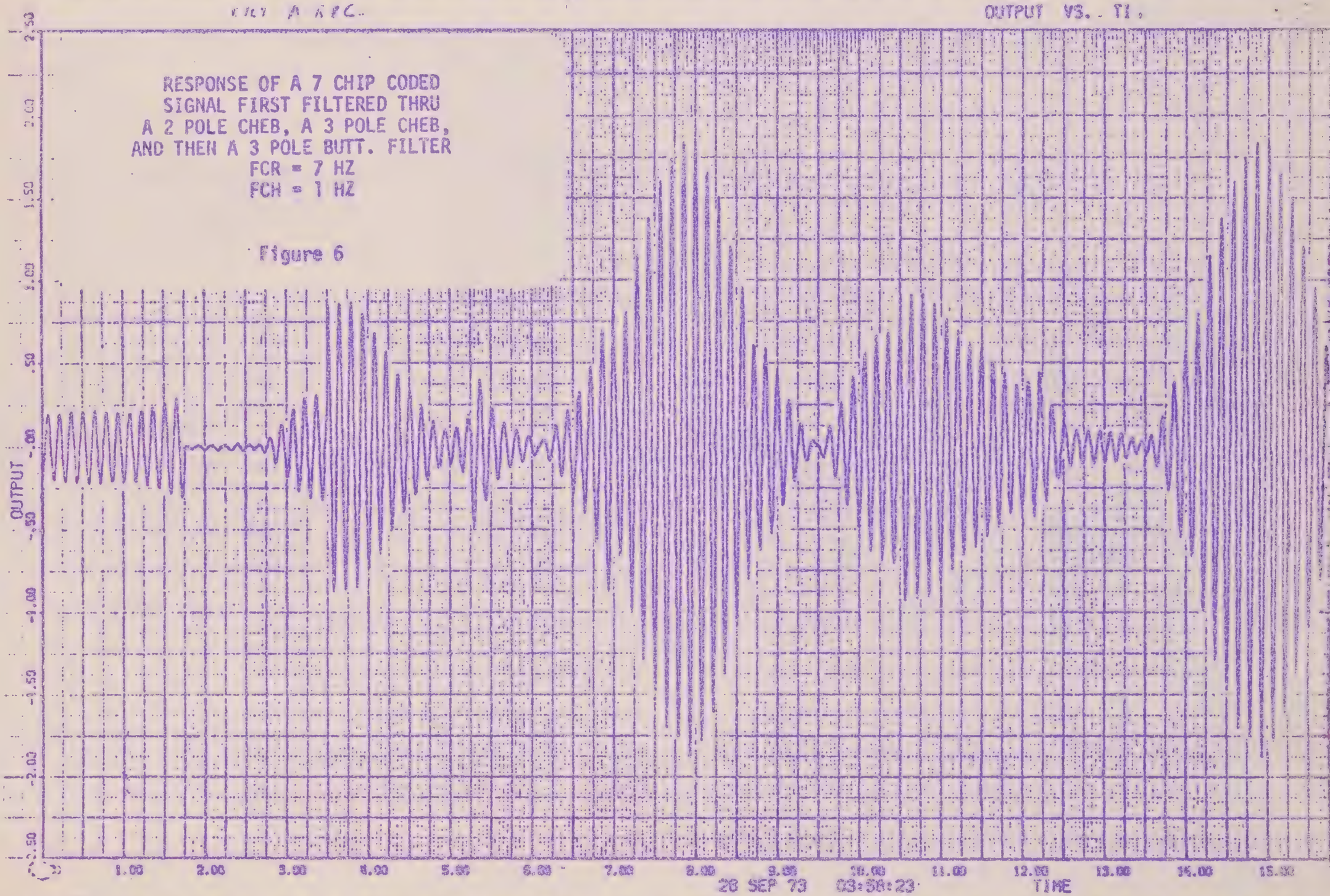
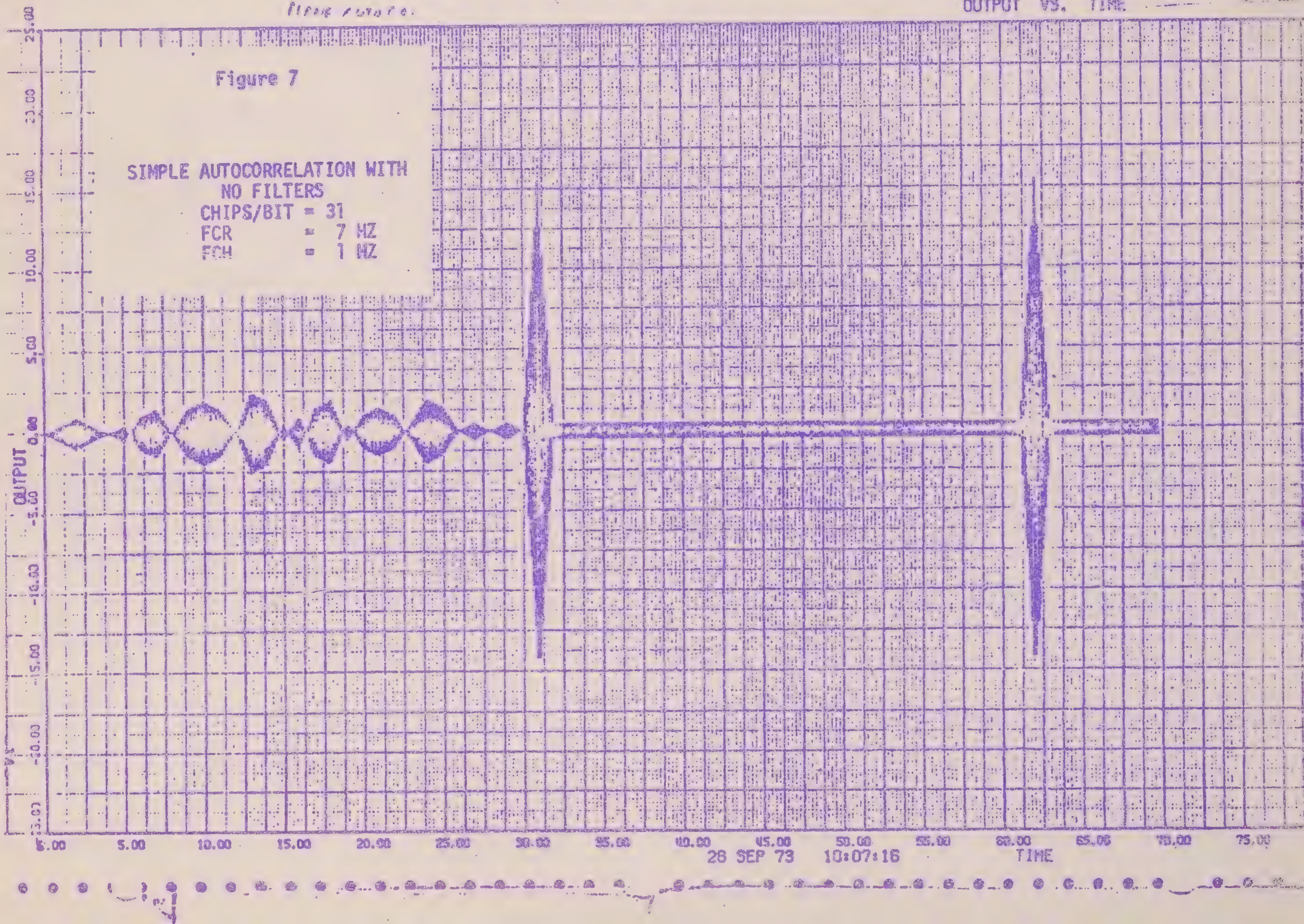

OUTPUT VS. TIME
CHIPS/BIT = 7
FCR = 7 HZ
FCH = 1 HZ
SIGNAL RESPONSE WITH A 3 POLE
CHEBYCHEV FILTER IN LINE
28 SEP 73 03:41:32
TIME

Figure 4


OUTPUT VS. TIME
CHIPS/BIT = 7
FCR = 7 HZ
FCH = 1 HZ
SIGNAL RESPONSE WITH A 3 POLE
BUTTERWORTH FILTER IN LINE
Figure 5
28 SEP 73 03:41:50
TIME


OUTPUT VS. TI.
RESPONSE OF A 7 CHIP CODED
SIGNAL FIRST FILTERED THRU
A 2 POLE CHEB, A 3 POLE CHEB,
AND THEN A 3 POLE BUTT. FILTER
FCR = 7 HZ
FCH = 1 HZ
Figure 6
OUTPUT
TIME
28 SEP 73 03:56:23


OUTPUT VS. TIME
Figure 7
SIMPLE AUTOCORRELATION WITH
NO FILTERS
CHIPS/BIT = 31
FCR = 7 MZ
FCH = 1 MZ
OUTPUT
25.00
20.00
15.00
10.00
5.00
0.00
-5.00
-10.00
-15.00
-20.00
-25.00
0.00
5.00
10.00
15.00
20.00
25.00
30.00
35.00
40.00
45.00
50.00
55.00
60.00
65.00
70.00
75.00
28 SEP 73 10:07:16
TIME

FIL 1 B

OUTPUT VS. TIME

CHIPS/BIT = 31
FCR = 7 HZ
FCH = 1 HZ
SIGNAL RESPONSE WITH A 3 POLE CHEBYCHEV FILTER IN LINE

Figure 9

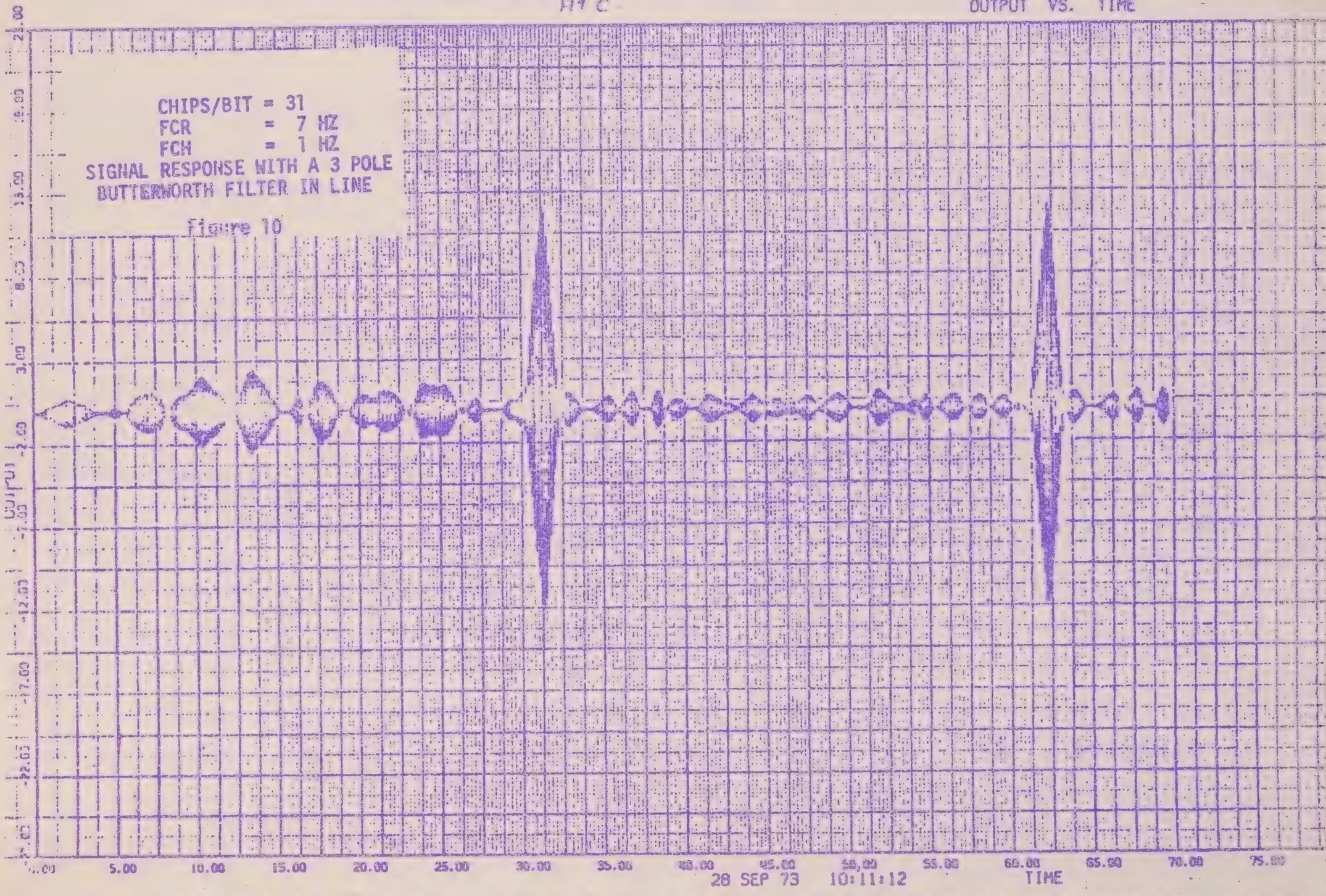
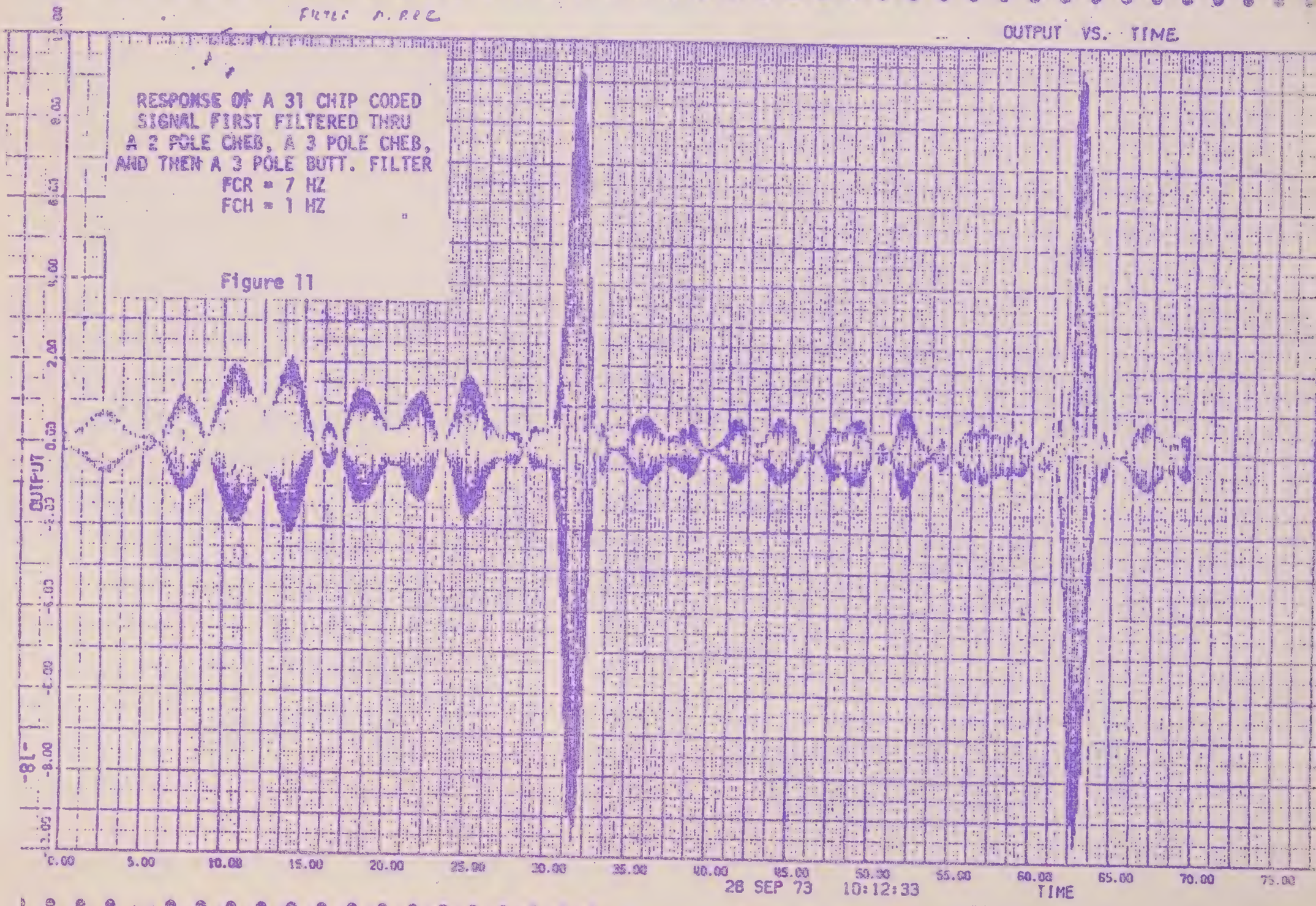

OUTPUT VS. TIME
CHIPS/BIT = 31
FCR = 7 HZ
FCH = 1 HZ
SIGNAL RESPONSE WITH A 3 POLE
BUTTERWORTH FILTER IN LINE
28 SEP 73 10:11:12
TIME

Figure 10


OUTPUT VS. TIME
RESPONSE OF A 31 CHIP CODED
SIGNAL FIRST FILTERED THRU
A 2 POLE CHEB, A 3 POLE CHEB,
AND THEN A 3 POLE BUTT. FILTER
FCR = 7 HZ
FCH = 1 HZ
Figure 11
OUTPUT
TIME
28 SEP 73 10:12:33

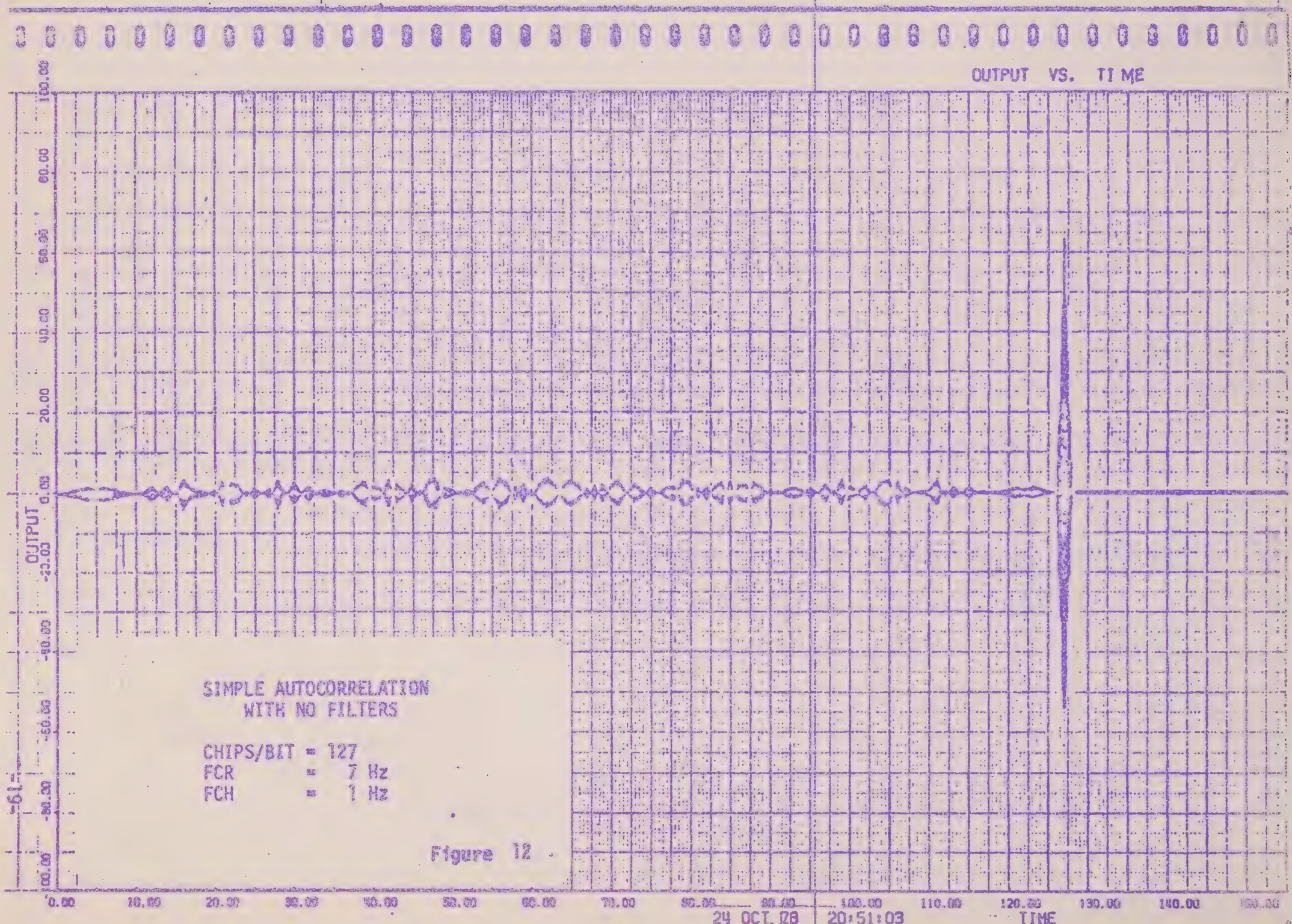
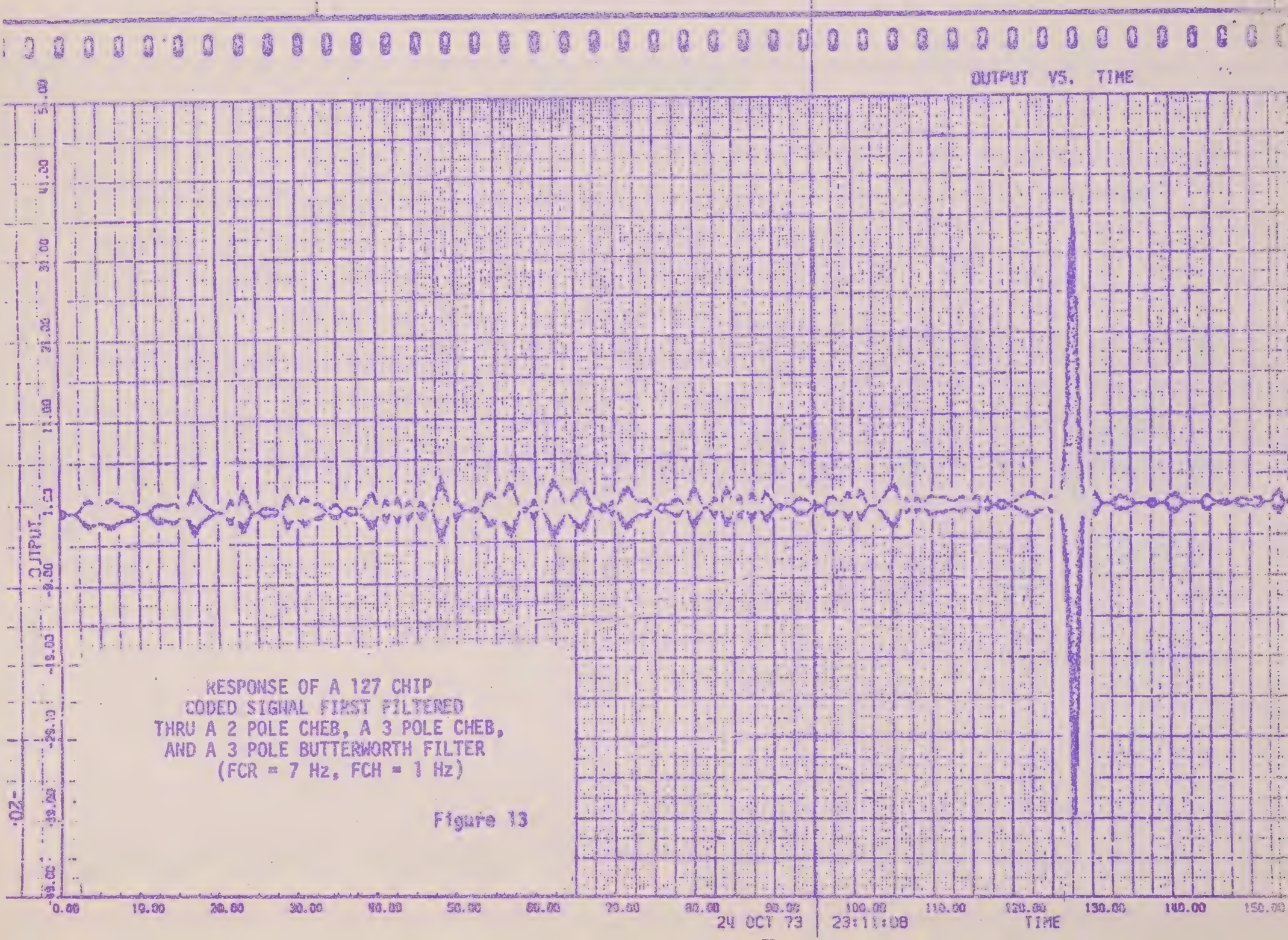
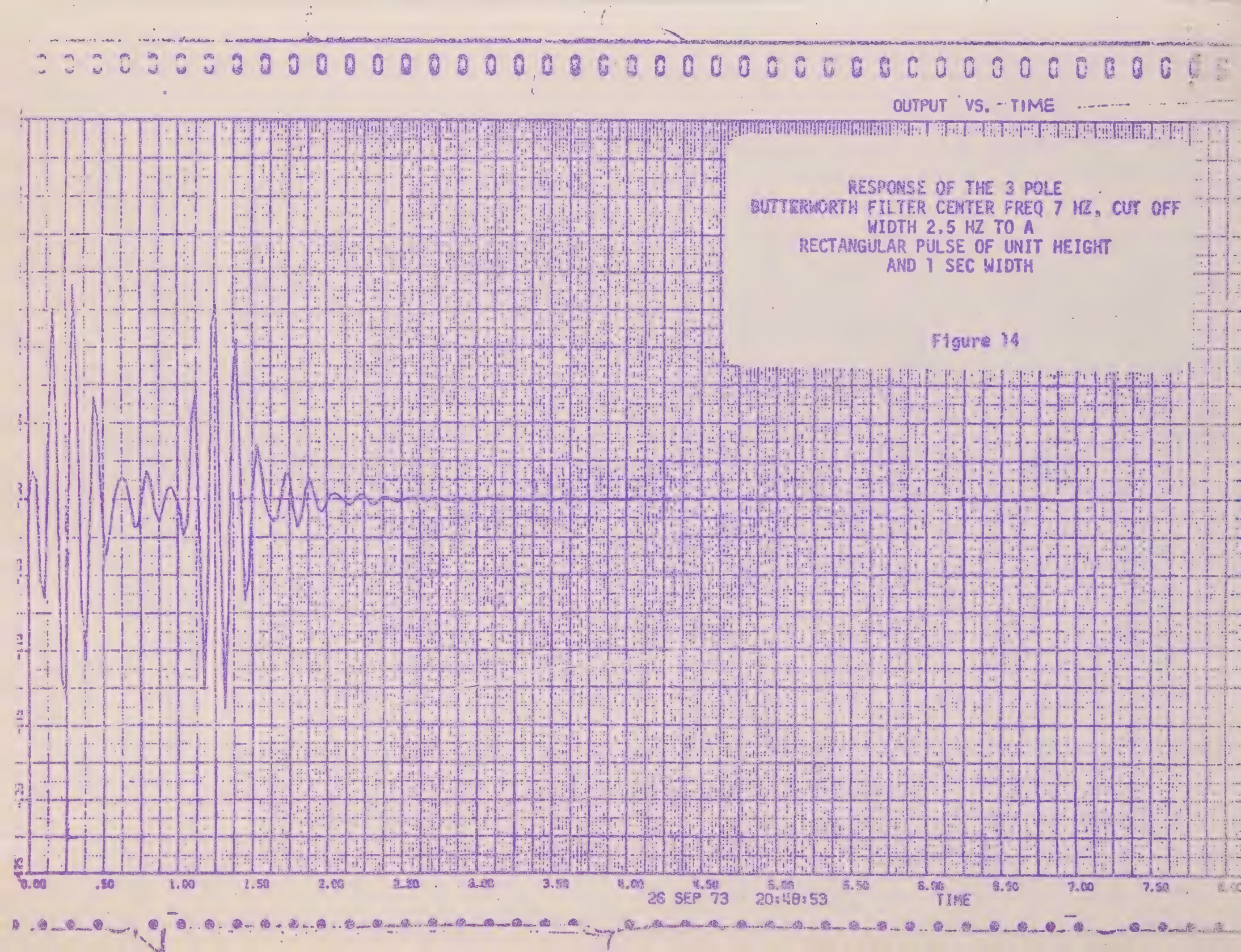

OUTPUT VS. TIME
OUTPUT
100.00
80.00
60.00
40.00
20.00
0.00
-20.00
-40.00
-60.00
-80.00
-100.00
SIMPLE AUTOCORRELATION
WITH NO FILTERS
CHIPS/BIT = 127
FCR = 7 Hz
FCH = 1 Hz
0.00
10.00
20.00
30.00
40.00
50.00
60.00
70.00
80.00
90.00
100.00
110.00
120.00
130.00
140.00
24 OCT 78
20:51:03
TIME

Figure 12



RESPONSE OF A 127 CHIP
CODED SIGNAL FIRST FILTERED
THRU A 2 POLE CHEB, A 3 POLE CHEB,
AND A 3 POLE BUTTERWORTH FILTER
(FCR = 7 Hz, FCH = 1 Hz)

Figure 13


OUTPUT VS. TIME
RESPONSE OF THE 3 POLE
BUTTERWORTH FILTER CENTER FREQ 7 HZ, CUT OFF
WIDTH 2.5 HZ TO A
RECTANGULAR PULSE OF UNIT HEIGHT
AND 1 SEC WIDTH
Figure 14
0.00
.50
1.00
1.50
2.00
2.50
3.00
3.50
4.00
4.50
5.00
5.50
6.00
6.50
7.00
7.50
26 SEP 73 20:48:53
TIME

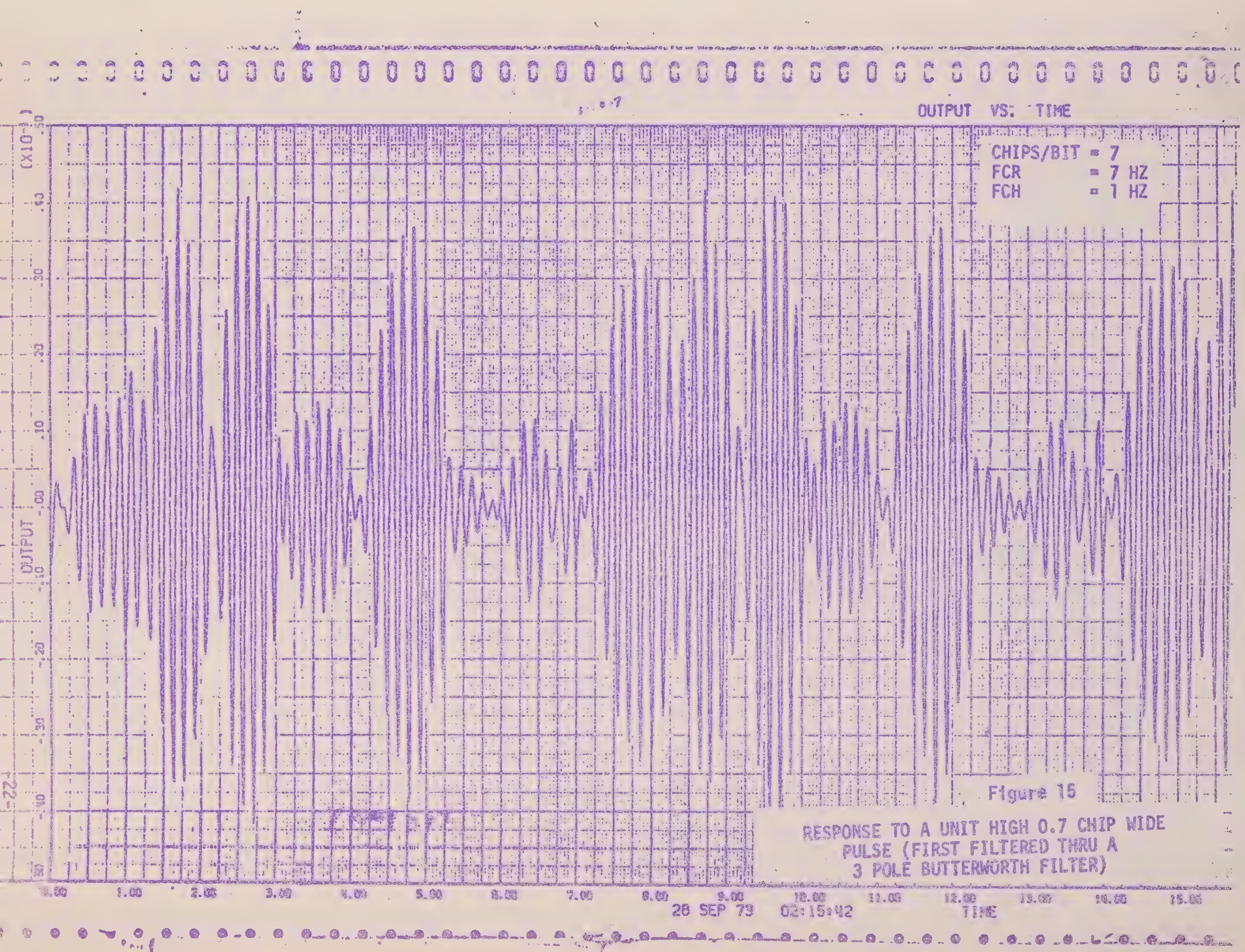


Figure 15

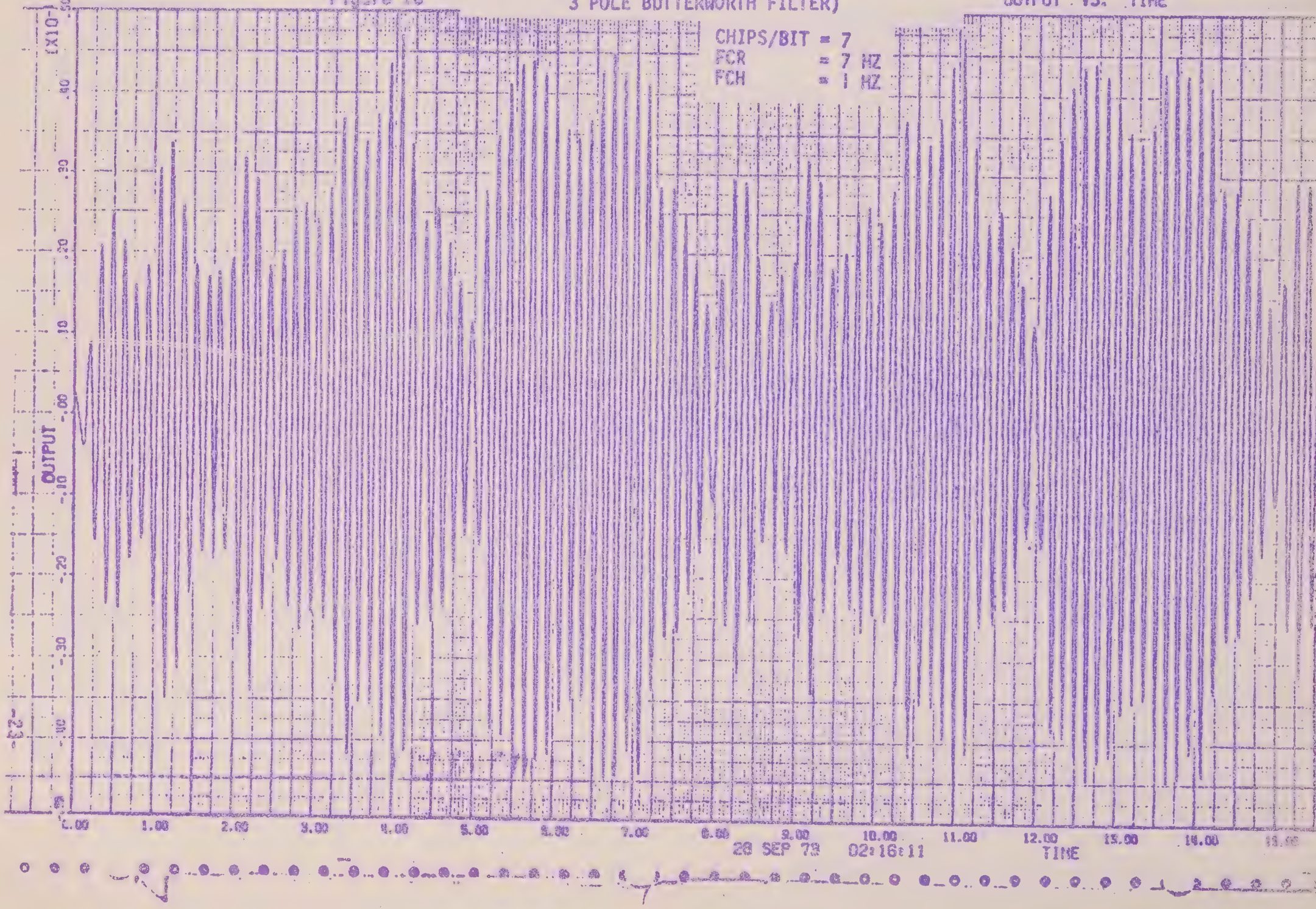
RESPONSE TO A UNIT HIGH 0.7 CHIP WIDE PULSE (FIRST FILTERED THRU A 3 POLE BUTTERWORTH FILTER)

Figure 16

RESPONSE TO A UNIT HIGH 0.8 CHIP WIDE PULSE (FIRST FILTERED THRU A 3 POLE BUTTERWORTH FILTER)

OUTPUT VS. TIME

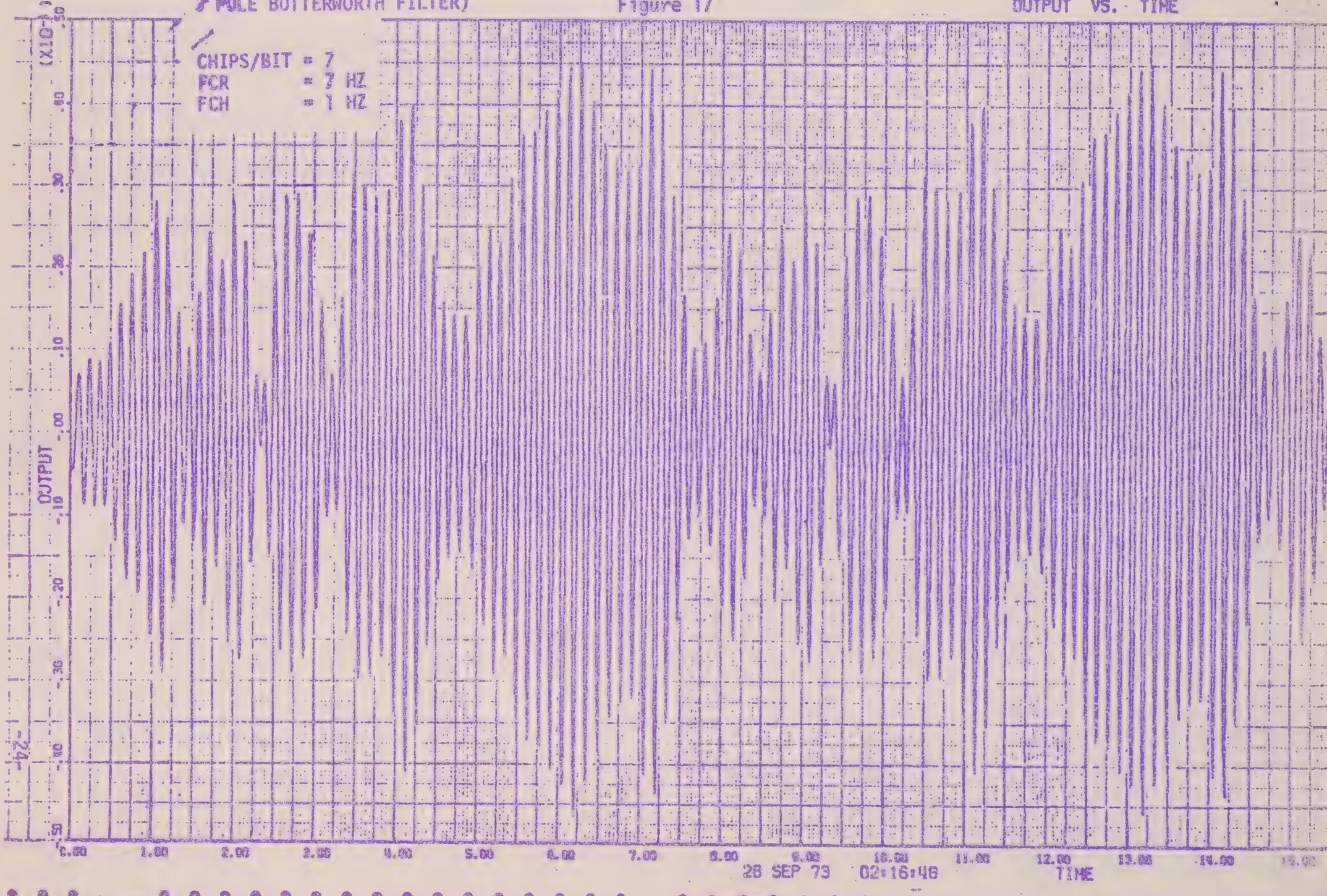
CHIPS/BIT = 7
FCR = 7 HZ
FCH = 1 HZ



RESPONSE TO A UNIT HIGH 0.9 CHIP WIDE PULSE (FIRST FILTERED THRU A 3 POLE BUTTERWORTH FILTER)

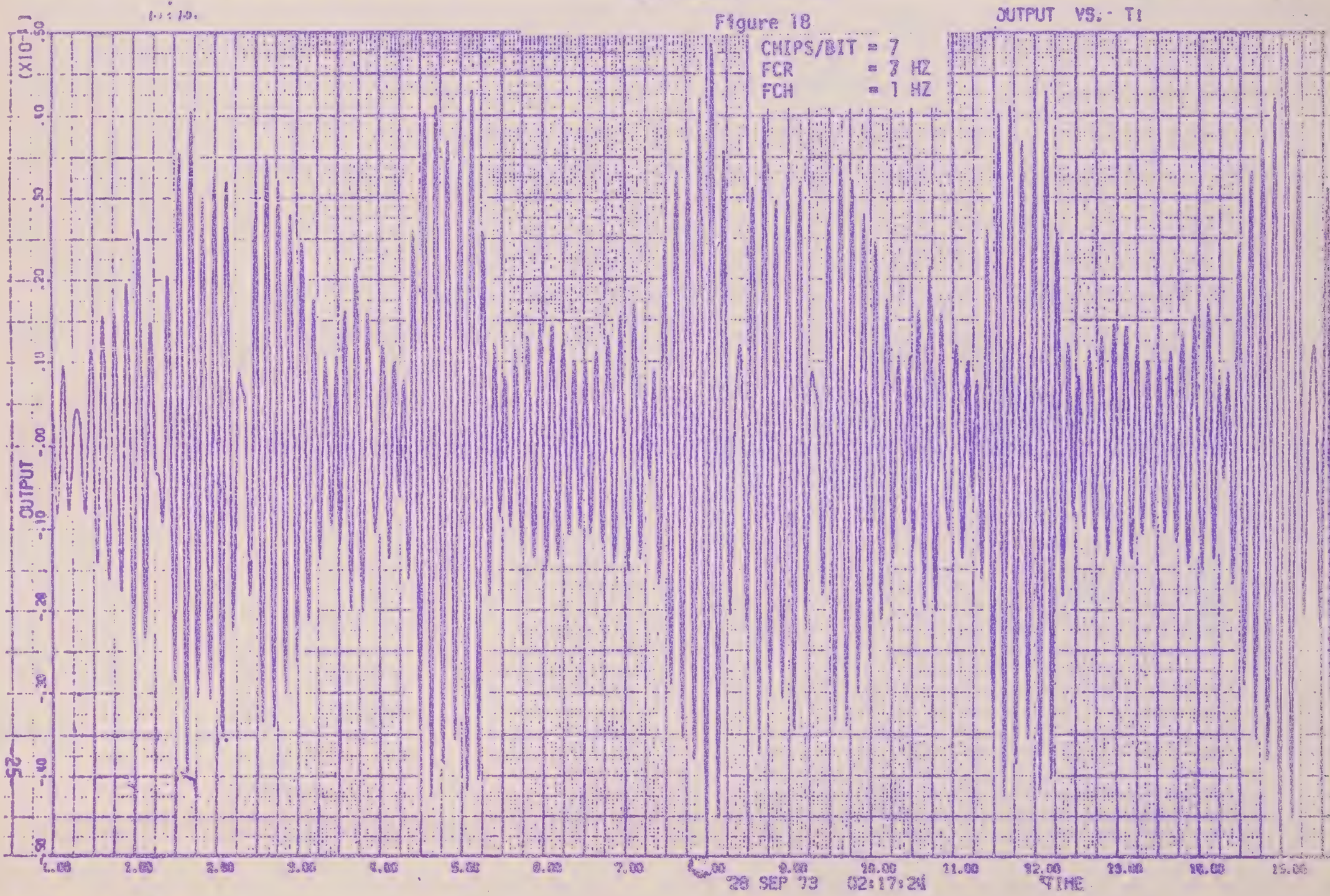
Figure 17

OUTPUT VS. TIME



RESPONSE TO A UNIT HIGH 1 CHIP WIDE PULSE (FIRST FILTERED THRU A 3 POLE BUTTERWORTH FILTER)

Figure 18



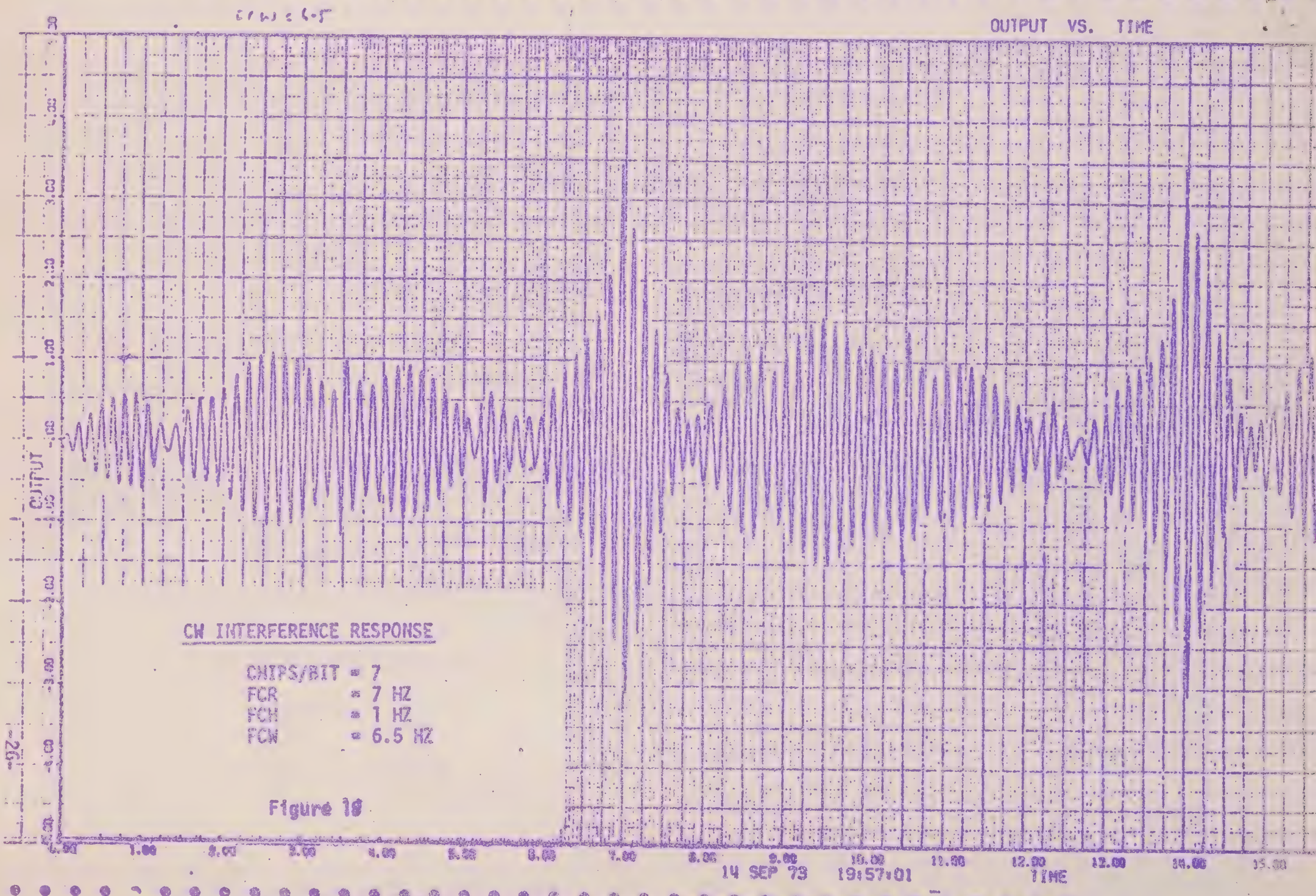

OUTPUT VS. TIME
OUTPUT
TIME
14 SEP 73 19:57:01
CW INTERFERENCE RESPONSE
CHIPS/BIT = 7
FCR = 7 HZ
FCH = 1 HZ
FCW = 6.5 HZ

Figure 18

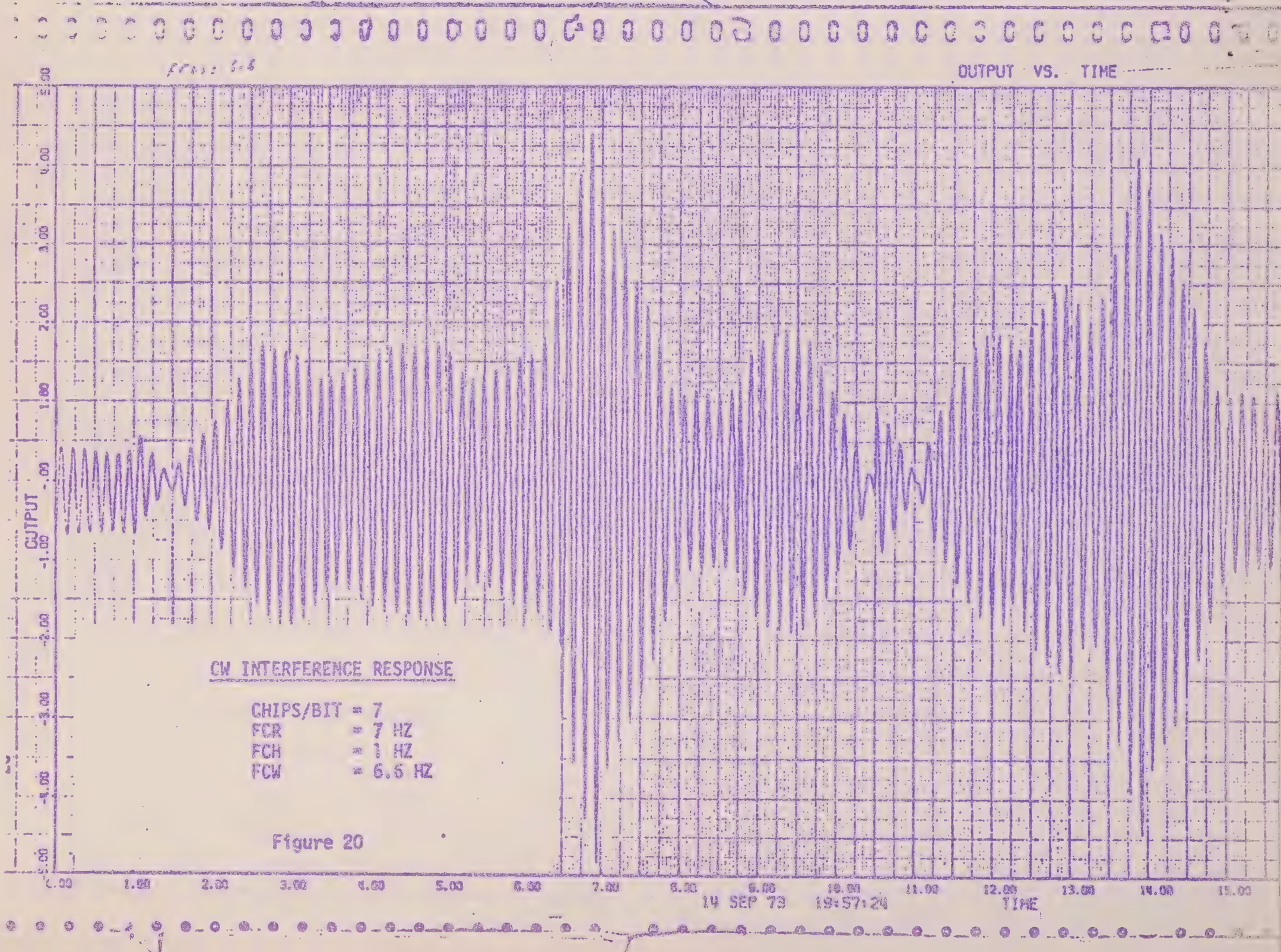

OUTPUT VS. TIME
OUTPUT
TIME
14 SEP 73 19:57:24
CW INTERFERENCE RESPONSE
CHIPS/BIT = 7
FCR = 7 HZ
FCH = 1 HZ
FCW = 6.6 HZ

Figure 20

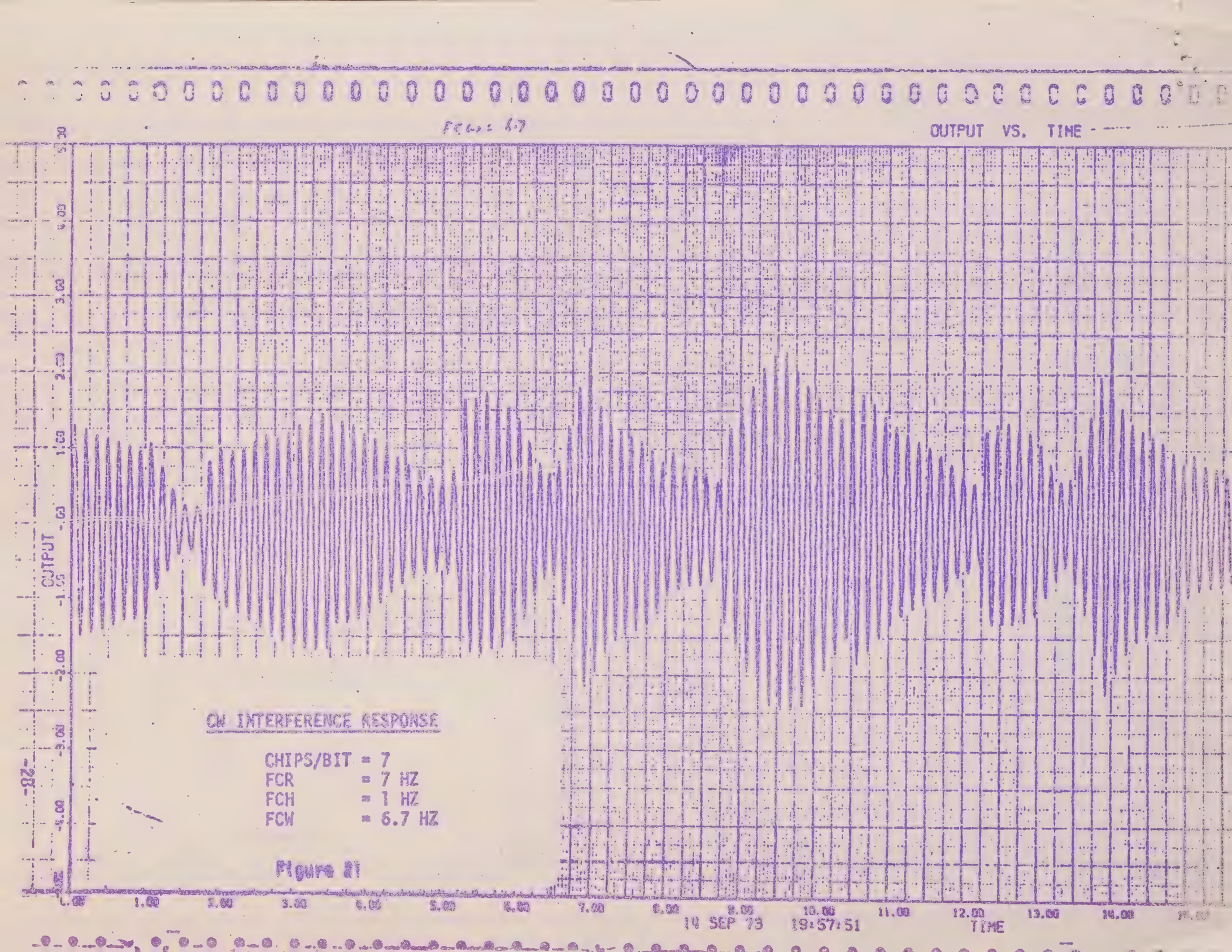

OUTPUT VS. TIME
OUTPUT
TIME
14 SEP 73 19:57:51
CW INTERFERENCE RESPONSE
CHIPS/BIT = 7
FCR = 7 HZ
FCH = 1 HZ
FCW = 6.7 HZ

Figure 21

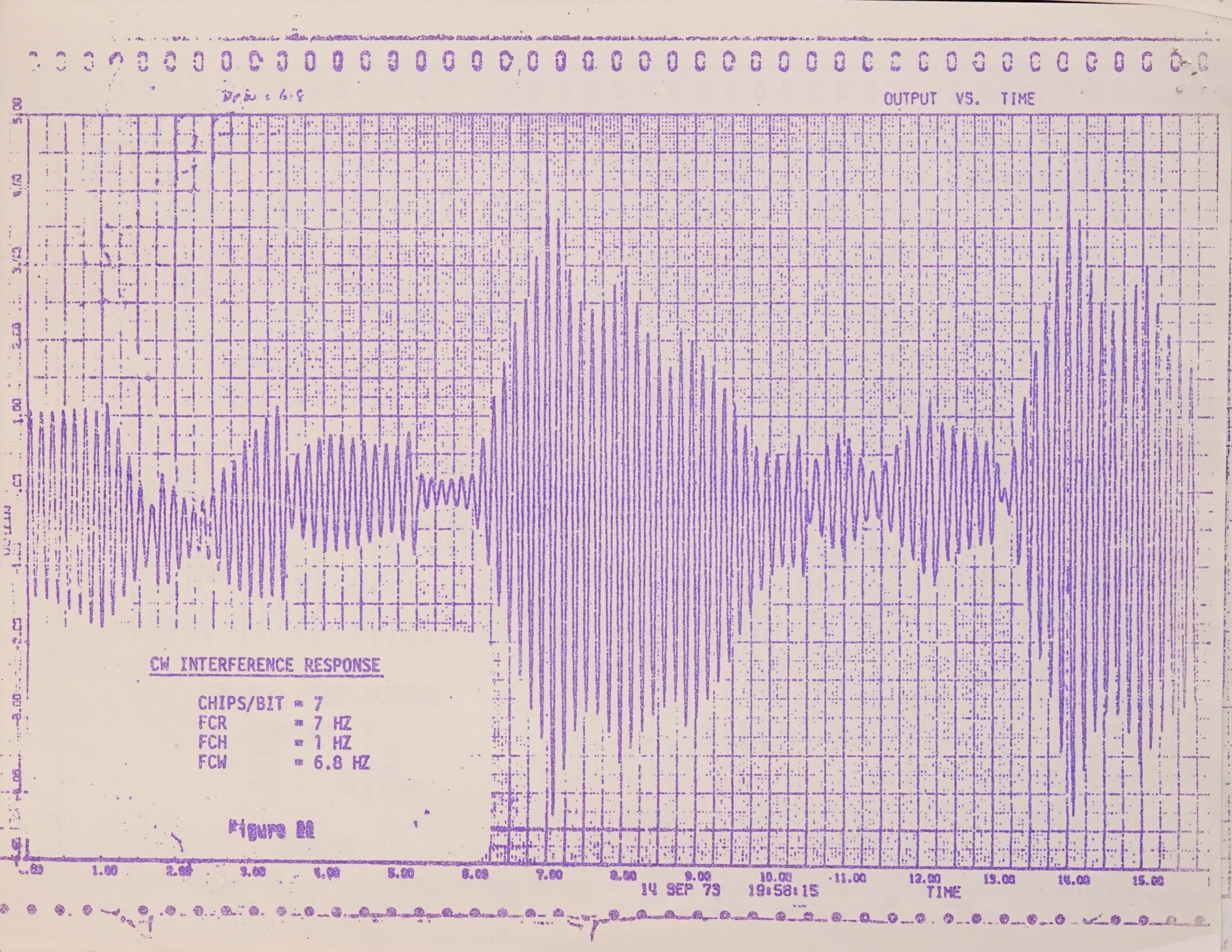
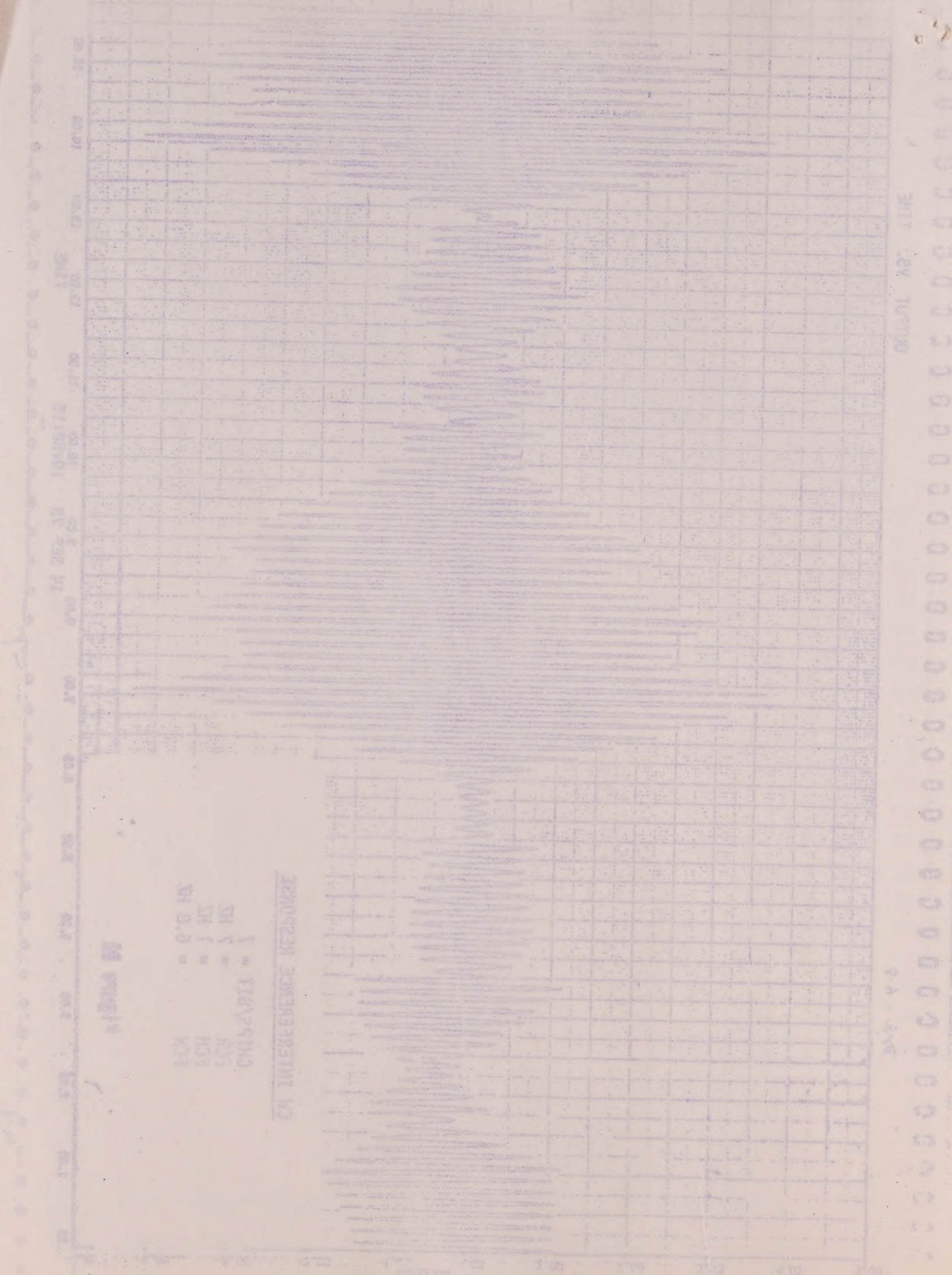

OUTPUT VS. TIME
CW INTERFERENCE RESPONSE
CHIPS/BIT = 7
FCR = 7 HZ
FCH = 1 HZ
FCW = 6.8 HZ
14 SEP 79 19:58:15
TIME

Figure 22



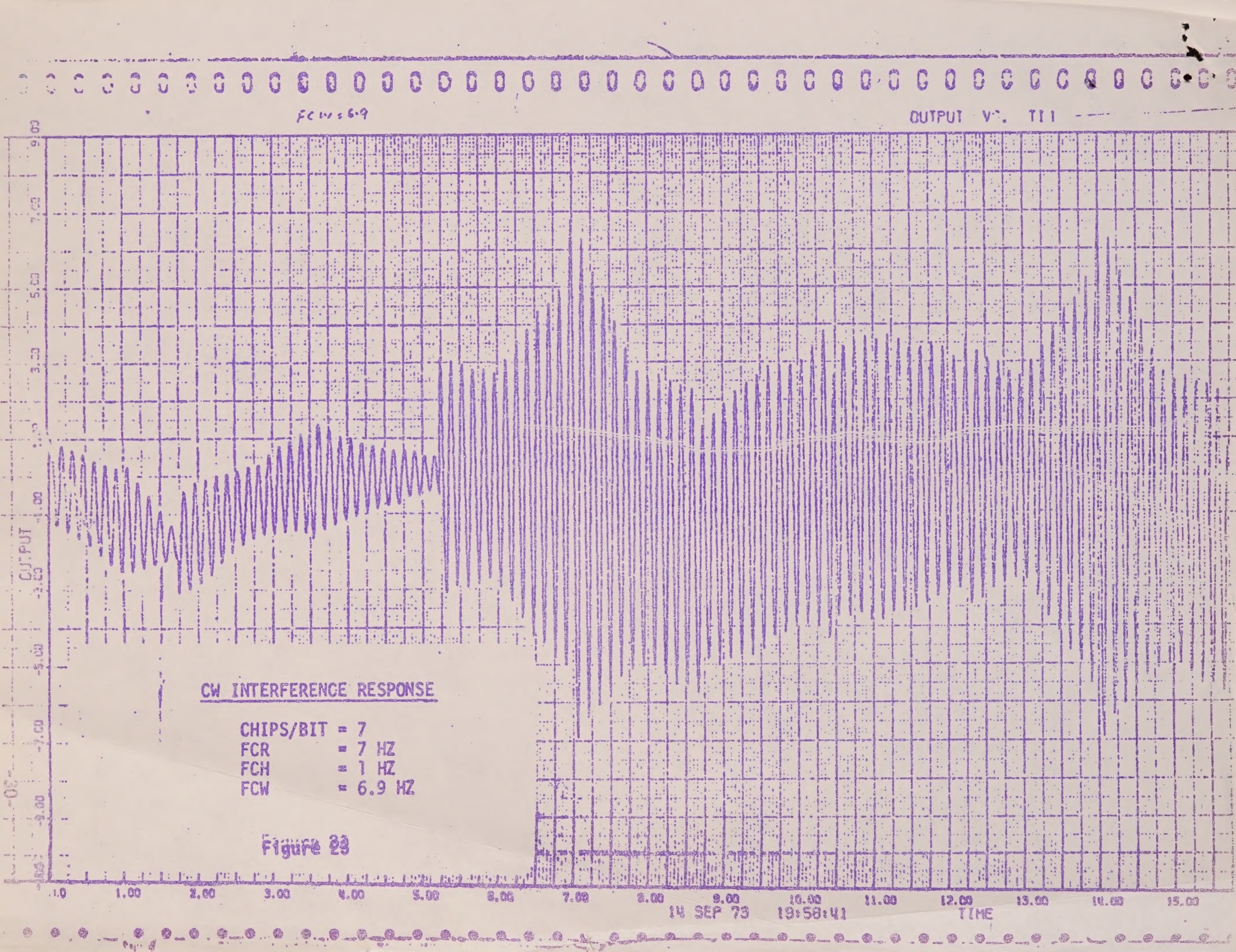

FCW = 6.9
OUTPUT V. TI I
OUTPUT
TIME
14 SEP 79 19:58:41
CW INTERFERENCE RESPONSE
CHIPS/BIT = 7
FCR = 7 HZ
FCH = 1 HZ
FCW = 6.9 HZ

Figure 23